Ancient Egyptian chronology and the astronomical orientation of pyramids

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The ancient Egyptian pyramids at Giza have never been accurately dated, although we know that they were built approximately around the middle of the third millennium BC. The chronologies of this period have been reconstructed from surviving lists of kings and the lengths of their reigns, but the lists are rare, seldom complete and contain known inconsistencies and errors. As a result, the existing chronologies for that period (the Old Kingdom) can be considered accurate only to about ±100 years, a figure that radiocarbon dating cannot at present improve. Here I use trends in the orientation of Old Kingdom pyramids to demonstrate that the Egyptians aligned them to north by using the simultaneous transit of two circumpolar stars. Modelling the precession of these stars yields a date for the start of construction of the Great Pyramid that is accurate to ±5 yr, thereby providing an anchor for the Old Kingdom chronologies.

The pyramids of the middle of the third millennium in Egypt (the Old Kingdom), built as tombs for the kings of the period, were oriented to the cardinal points with extraordinary precision. The most accurately aligned is the Pyramid of Khufu (Cheops) at Giza, also known as the Great Pyramid, the sides of which deviate from true north by an average of less than 3 minutes of arc[1]. Ever since modern survey techniques revealed this achievement in the late nineteenth century[2], the question of how the ancient Egyptians achieved such accuracy has been widely debated.

The absence of contemporary source material accounts for the range of possible orientation methods that have been proposed over the years. There are no relevant texts or representations from this period and discussions have therefore relied either on much later textual or representational evidence or on considerations of potential accuracy[3]. Although for many years it has been accepted on the grounds of accuracy that a stellar method was used[3,4], recent research has revived the possibility of solar alignment[5].

Until now, discussions of ancient Egyptian orientation methods have overlooked the evidence provided by the alignments of the pyramids themselves. Researchers have made the implicit assumption that the ancient method used for orientation had the potential to produce the same degree of accuracy at any period[3] but this assumption is not supported by the evidence. Lists of measurements of the alignment of pyramids (refs 3, 6) show that the alignment of the pyramid of Khufu represented a peak of accuracy which was not maintained in subsequent reigns. In fact, after Khufu's reign the alignment of pyramids became increasingly inaccurate. If the Egyptians had mastered a method of exceptionally accurate orientation, they should have been able to reproduce the results in subsequent generations. Nor can the accuracy of the Khufu alignment be considered coincidental when overall trends in pyramid orientation are examined in detail.

Table 1 lists the pyramids for which accurate measurements of orientation are available together with the accession dates of the kings for whom they were constructed. As a result of differences in reconstructions of the historical data, several chronologies of the period are available[7]; here I follow the lower range of dates given in von Beckerath's recent chronology[8], with the exception of the reign of Snofru. Stadelmann's proposed 46-year reign of Snofru is followed here[9], and the start dates for Snofru's construction of Meidum and the Bent and Red pyramids also follow his chronology for the reign[10].

I assume here that the pyramid alignment ceremony occurred in year 2 of each king's reign (with the exception of those for the later pyramids of Snofru), after the burial of his predecessor, the choice of a suitable location, and preparation and levelling of the site. This accords with the fact that even kings with short reigns are known to have started construction of their own tombs.

Table 1 Date and orientation of ancient Egyptian pyramids

Ruler	Currently accepted accession date	Orientation—west side of pyramid	Orientation—east side of pyramid	Recalibrated accession date
Djoser	2640 BC		~ + 180′ (ref. 3)	
Snofru–Meidum (1)	2600 BC (–2/+17)	–18.1′ (ref. 11) ±1.0	–20.6′ (ref. 11) ±1.0	2526 BC ± 7
Snofru–Bent Pyramid (2)	[2583 BC] (–2/+11)	–11.8′ (ref. 12) ± 0.2	–17.3′ (ref. 12) ± 0.2	
Snofru–Red Pyramid (3)	[2572 BC] (–2/+9)		–8.7′* ± 0.2	
Khufu (4)	2554 BC	–2.8′ (ref. 1) ± 0.2	–3. 4′ (ref. 1) ± 0.2	2480 BC ± 5
Khafre (5)	2522 BC (–1)	–6.0′ (ref. 1) ± 0.2	–6.0′ (ref. 1) ± 0.2	2448 BC ± 5
Menkaure (6)	2489 BC (–4)	Average: +14. 1′ ± 1.8′ (ref. 2)	+12.4′ (ref. 2) ± 1.0	2415 BC ± 10
Sahure (7)	2446 BC (–15)		~ –23′(ref. 3, 6) ± 10	2372 BC ± 25
Neferirkare (8)	2433 BC (–16)		~ + 30′ (ref. 3) ± 10	2359 BC ± 25
Unas	2317 BC	+ 17. 4′ (ref. 1)	+ 17.1′ (ref. 1)	
Senwosret I	1956 BC		~ –90′ (ref. 13)	
Amenemhat III	1853 BC		+ 15.7′ (ref. 14)	

* Accurate measurement of alignment provided by J. Dorner.

Numbers in parentheses after rulers' names refer to labelling in Figs 1 and 4. All measurements of orientation are in arcminutes. Error margins are given only for those structures that appear in Figs 1b and 4. The dates in column 2 are from von Beckerath's chronology (lower estimates)[8] with the exception of the length of Snofru's reign and the dates of construction of his pyramids (in square brackets) which follow Stadelmann[9,10]. Error margins (in parentheses) given for these dates reflect cumulative differences in reign length between existing chronologies and are calculated relative to the beginning of Khufu's reign. For Snofru's reign a range of 29 to 48 years is allowed, reflecting current debate[10,15]. For pyramids post-dating Khufu, the error margins reflect differences between two standard scholarly chronologies[8,16]. Dates plotted in Figs 1b and 4 are two years later than those tabulated (with the exception of the Bent and Red pyramids) as it is assumed here that alignment ceremonies took place in year 2 of each king's reign. Error margins for the orientation of the west and east sides of the pyramids are estimates based on the equipment used to measure the alignments. ±0.2 arcminutes is allowed for recent measurements taken with a meridian-seeking theodolite[1] and ±1 for those measured using a less accurate theodolite and star-sightings[2,11]. A nominal 10-arcminute allowance is made for the pyramids of Sahure and Neferirkare, the orientations of which were calculated from figures published in excavation reports[3]. The east side of Sahure's pyramid incorporates a surveying error, the size of which has been calculated from published measurements[6]. The final column shows dates recalibrated according to the method described in the text.

The dates and measurements of alignment for each pyramid are plotted in Fig. 1a. The figure shows that of the eight pyramids dating from 2600–2400 BC, six lie approximately in a straight line. The other two, the pyramids of Khafre and Sahure, lie close to this group. There is no clear relationship between this group of pyramids and those pre– and post-dating it: the pyramid of Djoser predates the first attempts to orient structures accurately to the cardinal points, whereas for the period from 2400–1800 BC we have only three structures for which measurements are available, too few to assess whether they form a pattern amongst themselves.

The eight pyramids constructed between the reigns of Snofru and Neferirkare are re-plotted in Fig. 1b. Researchers proposing stellar methods agree that the Egyptians used northern or circumpolar stars for orientation[1,3,4], which suggests that the alignment ceremony was carried out for either the east or west side of the pyramid (the central axis was probably never levelled, ruling out its orientation by this method). Where available, measurements of both east and west sides are marked. In some of the earlier pyramids there is considerable variation in the alignment of the different sides but this appears to be the result of difficulties in constructing right angles while laying out or extending the base. The graph suggests that only one side was accurately aligned and that it was the west side of the structure.

Figure 1b shows that if the pyramids of Khafre (number 5 in Fig. 1b) and Sahure (number 7) are temporarily ignored, the remaining six pyramids constructed between the reigns of Snofru and Neferirkare lie close to a straight line plotted through four of the points as a guide to the eye (line a). From this we can deduce that the pyramids were oriented by a method that varied in accuracy over time, becoming increasingly accurate until the reign of Khufu and then decreasing in accuracy at a steady rate. The fact that these six points converge closely on the line shows that the orientation technique could be applied with great precision despite the fact that the method itself was not accurate relative to true north.

Methods of orientation

These results are incompatible with any of the methods of orientation considered probable in current literature, solar or stellar, all of which involve the bisection of angles produced by measuring either sun shadows or two equivalent positions on the trajectory of a northern or circumpolar star (isolated at equal height or in its most easterly and westerly positions)[3–5]. These methods would have maintained a level of accuracy over time: graphically, all the measurements of alignment would lie in a band centred on true north. The width of the band would vary depending on the potential accuracy of the particular method and the points would be randomly scattered within this band. Experimental work has shown that a precise method of bisecting the angle between the most easterly and westerly positions of a northern star could potentially achieve an accuracy of within $\pm$ 3 arcminutes (J. Dorner, personal communication); however, only one measurement of actual pyramid alignment falls within a band of this width. Some of the more inaccurate methods proposed are likely to require error margins of more than $\pm$60 arcminutes.

In Fig. 1b, the apparent precision of the alignment method (shown by the proximity of the six points to line a) strongly suggests stellar orientation, as solar methods would be unlikely to produce such accurate results. This conclusion is supported by the progressive deviation of the alignments from true north, which could not be achieved using a solar orientation method. The most likely explanation of this deviation from north is that a stellar orientation method was used which became increasingly inaccurate as a result of the effects of precession.

The north celestial pole appears from the earth to be a point on the celestial sphere around which the stars rotate. This point is directly aligned with the axis of rotation of the earth. However, the revolving axis of the earth is itself unstable and rotates slowly, like a gyroscope. This is precession and, as a result, the north celestial pole appears to trace out a large circle on the northern sky, with each cycle taking around 26,000 years. This movement is extremely slow; without a system of recorded measurements it would be unnoticeable to observers, even over periods of hundreds of years.

Today, the position of the north celestial pole is marked approximately by the star α-Ursae Minoris (Polaris) which lies within one degree of the pole. However, during the period of pyramid construction there was no star accurately marking the pole. The closest star was α-Draconis but this lay nearly two degrees from the celestial pole at the beginning of the reign of Khufu. In any case, sightings

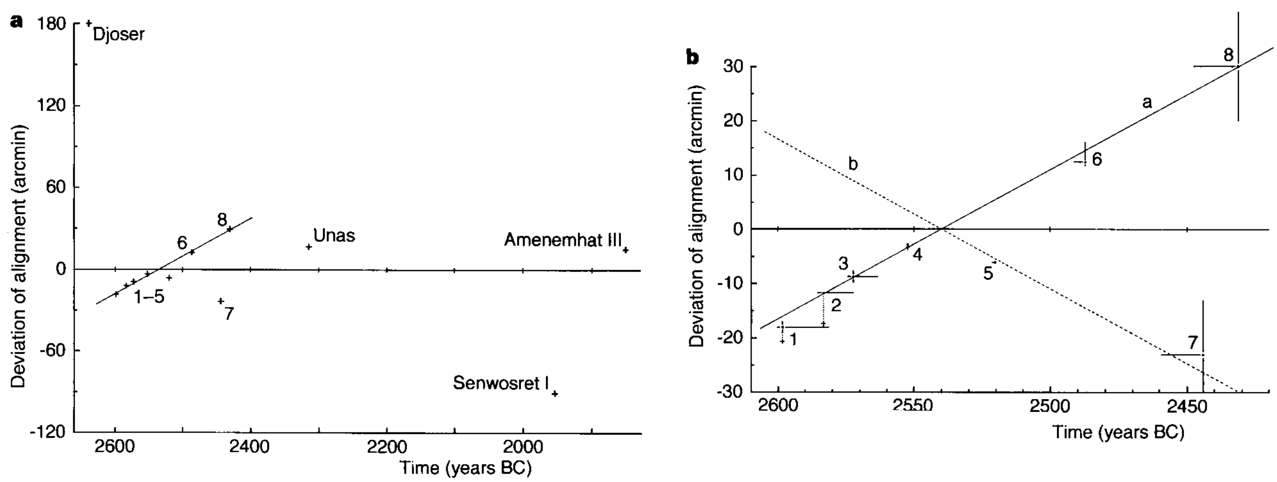

Figure 1 Deviation of pyramid alignments from true north over time. The dates plotted are 2 years later than the accession dates tabulated in Table 1 as it is assumed here that the alignment ceremony took place in year 2 of any given reign. 1, Meidum; 2, Bent Pyramid; 3, Red Pyramid; 4, Khufu; 5, Khafre; 6, Menkaure; 7, Sahure; 8, Neferirkare. **a**, 2640–1850 BC. Error bars are omitted as the uncertainties in some of the measurements of alignment are not well characterized; potential errors in the relative dating of the monuments are also difficult to assess over so large a period of time. **b**, 2600–2430 BC. West side alignments are plotted where available; east side alignments are marked as crosses below the west side measurement (numbers 1,2,4); additional error allowances are made for the Red Pyramid ($\pm$1) and Menkaure's pyramid (from calculations of the average and mean error of the other sides[2]) where no measurement of the west side alignment is available. Line a is plotted as a guide to the eye through the most recently and accurately surveyed points. If the simultaneous transit method was used to align the pyramids, the graph would form two lines of equal gradient, one positive and one negative, crossing the *y*-axis at 0. To show this, line b was derived from line a to be of equal but negative gradient; it passes close to the positions plotted for the pyramids of Khafre and Sahure. Pyramids lying on line a would be oriented with a star of Ursa Minor at its upper culmination and those on line b with a star of Ursa Major at its upper culmination.

taken toward a single star could not have achieved the results seen in Fig. 1b as, without a method of isolating culminations (the positions of greatest or least altitude of a star, which are also the points at which it crosses the meridian), the measurements would become increasingly random as the star moved away from the pole.

The method of orientation I propose is that the pole was considered to be located on an invisible chord linking two circumpolar stars on opposite sides of the pole. These two stars rotate around the pole, and when they are vertically aligned above the north horizon (one at its upper culmination and the other at its lower) an alignment made toward these stars with a plumb-line will be exactly oriented to true north, as long as the chord itself passes precisely through the pole. With the exception of the date when the chord between the stars lies exactly on the precessional trajectory of the pole, this method will produce alignments that become increasingly inaccurate at a steady rate over time. This method therefore has the potential to correspond with the results seen in Fig. 1b. This will be referred to as the simultaneous transit method because it uses two stars which cross (transit) the meridian simultaneously to establish true north.

Modelling the simultaneous transit method

A period from 2750 to 2350 BC (2550 BC ± 200 years, double the maximum error margin conventionally estimated for the relative chronologies of this period)[7] was examined for pairs of bright stars within 15 degrees from the pole which could have been used in the simultaneous transit method. Using SkyMap Pro 6 (ref. 17) it was established that only two pairs of stars were joined by chords which crossed the celestial pole during this period: ζ-Ursae Majoris and β-Ursae Minoris (around 2467 BC), and ε-Ursae Majoris and γ-Ursae Minoris (around 2443 BC).

Figure 2a shows the circumpolar stars in 2467 BC, when a chord between ζ-Ursae Majoris and β-Ursae Minoris crossed the pole (the discrepancy between this date and the date of greatest precision of pyramid alignment shown in Fig. 1b will be discussed below). Figure 2b shows a view of the north horizon at Giza for the same date, showing the stars when they are vertically aligned (in simultaneous transit). A measurement of alignment taken with a plumb-line toward the stars at this time would be oriented exactly to true north.

The Khafre and Sahure alignments

The simultaneous transit method of alignment can also be used to explain the anomalous orientations of the pyramids of Khafre and Sahure, the two pyramids which do not conform with the trend set by the six other pyramids of the period. From Fig. 1b it is clear that although the deviation of these pyramids' alignment (numbers 5 and 7) lies west rather than east of north, they are approximately of the magnitude to be expected for their dates.

The simultaneous transit method involves making an alignment toward two stars when they are vertically aligned. The alignment can be taken when either of the stars is in the upper position. Because the upper culminations of the two stars fall approximately twelve hours apart, with the time of culmination of each star moving slowly through 24 hours in the course of a year, for about half the year the vertical alignment that falls within the hours of darkness will have one of the stars always above, and for the other half of the year the second star will be in the position of upper culmination when the two stars are vertically aligned during the course of the night.

When the chord between these two stars passes precisely through the celestial pole, an alignment taken toward the stars when they are

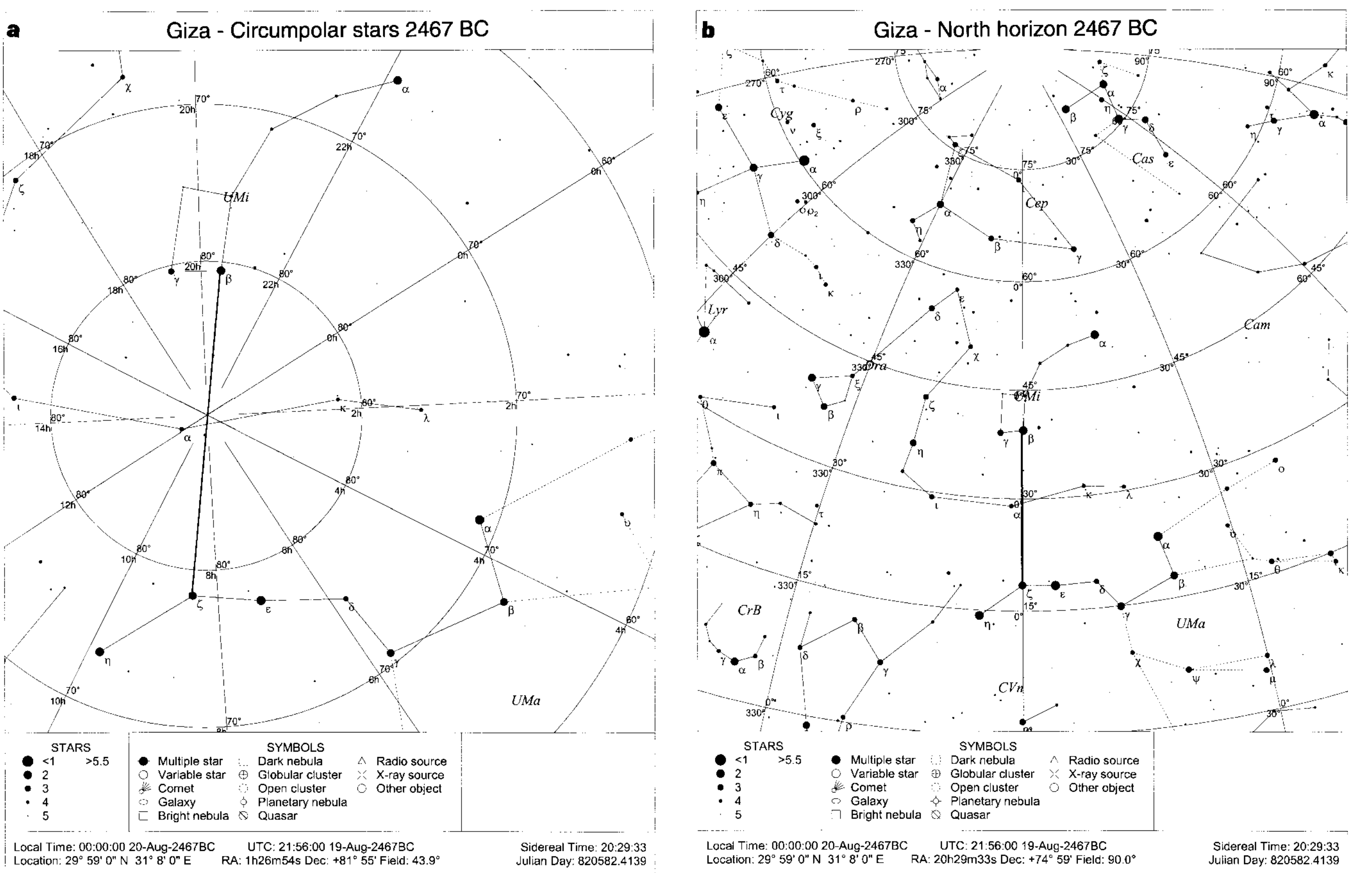

Figure 2 Modelling the simultaneous transit method for Giza, 2467 BC. **a**, A chord between stars β-UMi and ζ-UMa passes exactly through the north celestial pole. **b**, The same stars in simultaneous transit. An alignment taken toward these stars using a plumb-line would be oriented exactly to true north. Maps produced on *SkyMap Pro* 6 (ref. 17).

in simultaneous transit will be exact with either star at its upper culmination. However, as the pole moves away from the chord between the stars as a result of precession, the chord itself begins to rotate around the pole. Alignments made using the simultaneous transit method will deviate from true north at a rate which increases steadily over time, regardless of which star is at its upper culmination when the alignment was made. However, as the chord itself is rotating around the pole, alignments made with a particular star at its upper culmination will be the opposite side of the north pole to alignments taken when the same star is at its lower culmination. This is illustrated in Fig. 3.

A graph of alignments made using the simultaneous transit method should therefore take the form of two lines of equal gradient, one positive and one negative, crossing at true north. In Fig. 1b, the gradient of line a is used to generate a second line, b, of equal but negative gradient, crossing line a at zero on the y-axis. This line passes close to the points plotted for the pyramids of Khafre and Sahure.

The alignment of Khafre's and Sahure's pyramids can therefore be shown to have been generated by the same method of alignment as the other six pyramids of the period, the only difference being the time of year at which the orientation ceremony was carried out, and therefore the star which was uppermost. To show this graphically, the pyramids of Khafre and Sahure can be re-plotted on the opposite side of true north (Fig. 4). Line a passes through all four points for which recent accurate measurements are available (numbers 2–5), through the error bars of the three later points and close to the point for number 1.

Modelling the results

In order to test further the theory that the simultaneous transit method was used to orient this group of pyramids, the method was modelled mathematically. F. R. Stephenson (personal communication) calculated the distance from the pole of a chord running between the stars ζ-Ursae Majoris and β-Ursae Minoris. He confirmed that the chord ran precisely through the pole in 2467 BC (astronomical date −2466) by computing the date when the right ascension of the two stars differed by exactly 180 degrees. The same procedure was repeated for the pair of stars ϵ-Ursae Majoris and γ-Ursae Minoris which produced the date 2443 BC (astronomical date −2442). He then computed the minimum angular distance between the north celestial pole and the great circle passing through the pairs of stars at 25-year intervals around these dates. The results are plotted in Fig. 4 (lines b and c) alongside the graph derived from archaeological data (line a).

Figure 4 shows that measurements of the angular distance of the pole from a chord running between stars ζ-Ursae Majoris and β-Ursae Minoris produce a line (b) of very similar gradient to that implied by the archaeological data. Stars ϵ-Ursae Majoris and γ-Ursae Minoris produce a significantly steeper gradient (line c) which, crucially, cannot be accommodated to the three Giza pyramids (numbers 4–6) which form the most accurately fixed group temporally and spatially. These results strongly support the hypothesis that the pyramids were aligned using the simultaneous transit method, which was carried out using stars ζ-Ursae Majoris and β-Ursae Minoris. The only major discrepancy between the results produced by the two independent data sets is time: the line

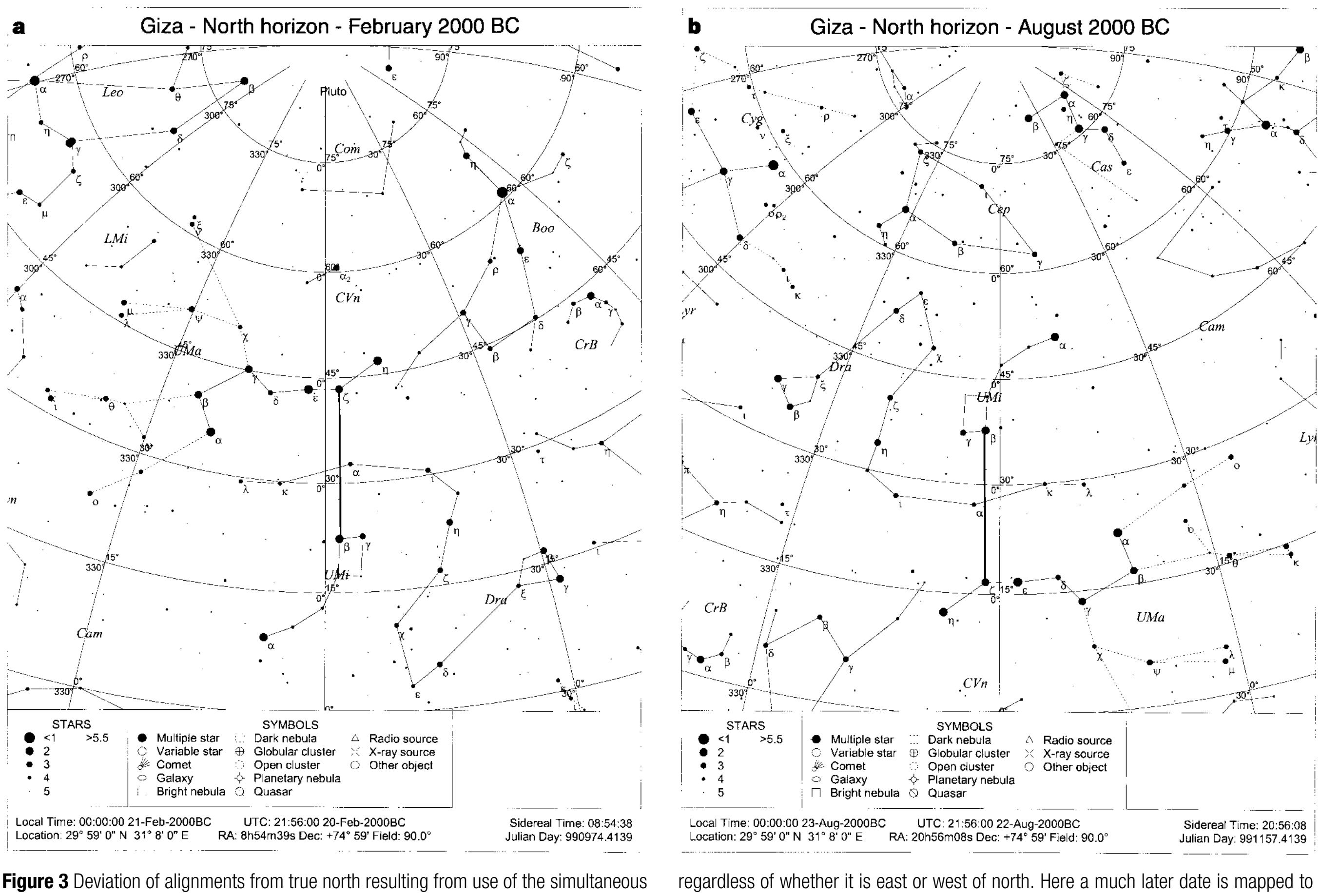

Figure 3 Deviation of alignments from true north resulting from use of the simultaneous transit method of orientation. As the chord between the pair of stars moves away from the pole, alignments taken toward the stars when they are in simultaneous transit (actually vertically aligned) may be on either side of the pole depending on which star is at its upper culmination. The size of the deviation from north will be the same at a given date regardless of whether it is east or west of north. Here a much later date is mapped to exaggerate the effect of time on the result of the alignment process. **a**, Winter and spring alignments with ζ-UMa at upper culmination; resulting alignments are east of north. **b**, Summer and autumn alignments with β-UMi at upper culmination; resulting alignments are west of north. Maps produced on *SkyMap Pro* 6 (ref. 17).

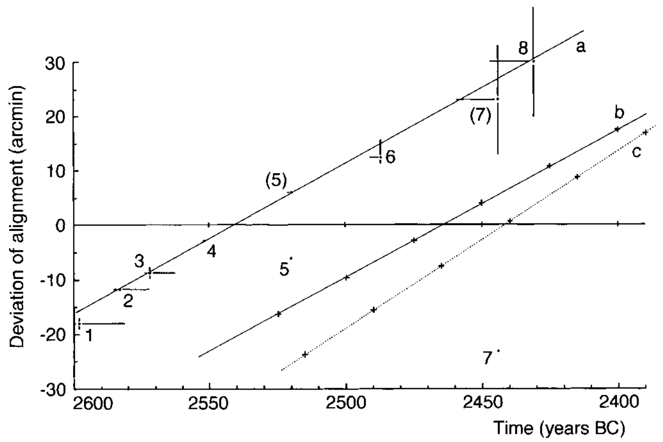

Figure 4 Astronomical modelling of the simultaneous transit method of orientation. Line a shows the deviation of pyramid alignment over time based on archaeological data plotted according to currently accepted chronologies of the period. Lines b and c plot the minimum angular distance between the north celestial pole and the great circle passing through pairs of stars which could have been used for simultaneous transit orientation (calculations by F. R. Stephenson). Line b uses stars β-UMi and ζ-UMa, which produce a gradient very similar to that implied by the archaeological data; line c uses stars γ-UMi and ε-UMa which produce a significantly steeper gradient. 1, Meidum; 2, Bent Pyramid; 3, Red Pyramid; 4, Khufu; 5, Khafre; 6, Menkaure; 7, Sahure; 8, Neferirkare. Parenthesized numbers (5 and 7) denote points re-plotted with positive rather than negative values, as described in the text, to conform to the dominant trend of the alignments.

generated by the astronomical modelling (line b) is over 70 years later than that derived from the archaeological results (line a).

Anchoring existing chronologies

The dating discrepancy between the two sets of results is caused by the fact that while stellar positioning at a given time can be predicted with great precision, existing Egyptian chronologies of this period based primarily on cumulative reign lengths can only be considered accurate to about ±100 years[7].

In Fig. 4, although the chronology of line b generated by astronomical data can be considered fixed, the chronology according to which the archaeological data are plotted (line a) is not anchored in time. However, the point at which line a crosses zero on the y-axis can now be fixed at 2467 BC from the results of the astronomical modelling. This gives a date of 2478 BC for the alignment of Khufu's pyramid which would require the lowering of von Beckerath's lower estimate of chronology by a further 74 years.

In reconstructing accession dates from dates of pyramid alignment ceremonies, potential for error theoretically exists in the assumption made here that this ceremony was held in the second year of each reign. However, it is exceptionally unlikely in this period that the error involved is more than ±1 year. At present, a total of ±5 years can be considered an adequate error allowance for the reigns of Khufu and Khafre (F. R. Stephenson and T. van Albada, personal communications) given the accuracy of the archaeological data available for these reigns and the precision of the astronomical modelling.

Future research

The ability to fix the reigns of Khufu and Khafre to ±5 years represents an advance in establishing a reliable absolute chronology for the second half of the third millennium BC in Egypt, but it does not solve all the problems. It is not possible simply to shift existing chronologies forward by the requisite number of years as fixed astronomical dates soon after 2000 BC mean that these existing chronologies will also have to be compressed. To achieve this, a process of careful reanalysis of the historical data will be necessary to make suitable adjustments. For this reason, the recalibrated accession dates given in the last column of Table 1 show error margins which increase over time as the possibility of numerous minor errors in cumulative reign lengths is compounded.

I intend to undertake fieldwork to collect more accurate data for those pyramids that have not been recently and reliably surveyed. From this and through more detailed mathematical modelling I hope to refine the error margin for dating the pyramids of Khufu and Khafre to ±1–2 years. More accurate data for the period around the reigns of Sahure and Neferirkare will reduce the error margins for the dates of these later kings and will assist in the process of refining the overall chronology of the period. □

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